



THE UNIVERSITY *of* TEXAS

---

HEALTH SCIENCE CENTER AT HOUSTON  
SCHOOL *of* HEALTH INFORMATION SCIENCES

# Solutions HW 1

For students of HI 6001-100 “Biomolecular Modeling”

Willy Wriggers, Ph.D.

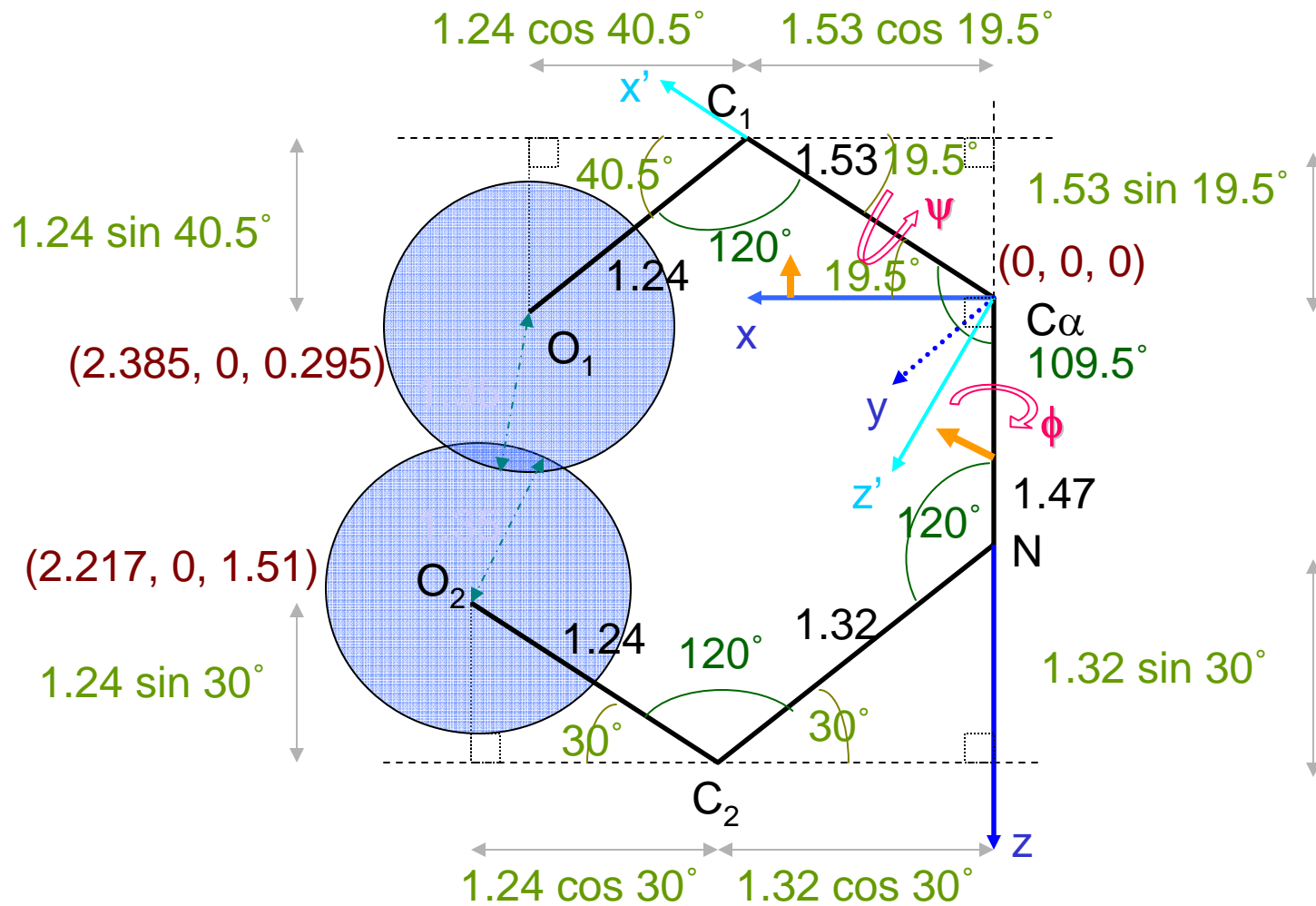
School of Health Information Sciences

<http://biomachina.org/courses/modeling/06.html>

# Problem 1

Amino Acid	Structure	pKa	H+ on	H+ off	pH 2.0	pH 7.3	pH13.0
Alanine	A $\text{H}_3\text{N}^+ \text{CH}(\text{CH}_3)\text{COOH}$	8.0	+1	0	+1	+1	0
Alanine	A $\text{CH}_3\text{CH}(\text{NH}_2)\text{COOH}$						
Valine	V $\text{H}_3\text{C}-\text{CH}(\text{H}_3\text{C})-\text{CH}(\text{NH}_2)\text{COOH}$						
Leucine	L $\text{H}_3\text{C}-\text{CH}(\text{H}_3\text{C})-\text{CH}_2-\text{CH}(\text{NH}_2)\text{COOH}$						
Isoleucine	I $\text{H}_3\text{C}-\text{CH}_2-\text{CH}(\text{H}_3\text{C})-\text{CH}(\text{NH}_2)\text{COOH}$						
Threonine	T $\text{H}_3\text{C}-\text{CH}(\text{HO})-\text{CH}(\text{NH}_2)\text{COOH}$						
Serine	S $\text{HO}-\text{CH}_2-\text{CH}(\text{NH}_2)\text{COOH}$						
Glutamine	Q $\text{H}_2\text{N}-\text{C}(=\text{O})-\text{CH}_2-\text{CH}_2-\text{CH}(\text{NH}_2)\text{COOH}$						
Asparagine	N $\text{H}_2\text{N}-\text{C}(=\text{O})-\text{CH}_2-\text{CH}(\text{NH}_2)\text{COOH}$						
Lysine	K $\text{H}_2\text{N}-(\text{CH}_2)_4-\text{CH}(\text{NH}_2)\text{COOH}$	10.0	+1	0	+1	+1	0
Arginine	R $\text{HN}(\text{NH}_2)-\text{CH}_2-\text{CH}_2-\text{CH}_2-\text{CH}(\text{NH}_2)\text{COOH}$	12.0	+1	0	+1	+1	0
Aspartate	D $\text{HOOC}-\text{CH}_2-\text{CH}(\text{NH}_2)\text{COOH}$	4.4	0	-1	0	-1	-1
Glutamate	E $\text{HOOC}-\text{CH}_2-\text{CH}_2-\text{CH}(\text{NH}_2)\text{COOH}$	4.4	0	-1	0	-1	-1
Histidine	H $\text{Imidazole ring}-\text{CH}_2-\text{CH}(\text{NH}_2)\text{COOH}$	6.5	+1	0	+1	0	0
Phenylalanine	F $\text{Phenyl ring}-\text{CH}_2-\text{CH}(\text{NH}_2)\text{COOH}$						
Tyrosine	Y $\text{HO}-\text{C}_6\text{H}_4-\text{CH}_2-\text{CH}(\text{NH}_2)\text{COOH}$	10.0	0	-1	0	0	-1
Tryptophan	W $\text{Indole ring}-\text{CH}_2-\text{CH}(\text{NH}_2)\text{COOH}$						
Methionine	M $\text{H}_3\text{C}-\text{S}-(\text{CH}_2)_2-\text{CH}(\text{NH}_2)\text{COOH}$						
Cysteine	C $\text{HS}-\text{CH}_2-\text{CH}(\text{NH}_2)\text{COOH}$	8.5	0	-1	0	0	-1
Proline	P $\text{Cyclic secondary amine ring}-\text{COOH}$						
Glycine	G $\text{H}-\text{CH}(\text{NH}_2)\text{COOH}$						
Alanine	A $\text{CH}_3\text{CH}(\text{NH}_2)\text{COOH}$						
Alanine	A $\text{CH}_3\text{CH}(\text{NH}_2)\text{COO}^-$	3.1	0	-1	0	-1	-1
<b>Net charge</b>			pH<pKa	pH>pKa	+4	0	-5

# Problem 2



All length units are in Å (0.1nm)

Rotating  $O_2$  by  $\phi$  about  $C\alpha-N$  (z axis):

$$O_2' = Rz(\phi) \cdot \begin{bmatrix} 2.217 \\ 0 \\ 1.51 \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2.217 \\ 0 \\ 1.51 \end{bmatrix}$$

$$= \begin{bmatrix} 2.217 \cos \phi + 0 + 0 \\ 2.217 \sin \phi + 0 + 0 \\ 0 + 0 + 1.51 \end{bmatrix}$$

$$= \begin{bmatrix} 2.217 \cos \phi \\ 2.217 \sin \phi \\ 1.51 \end{bmatrix}$$

*Double-check:*

*Based on dihedral angle definition, rotation by small positive  $\Phi$  means  $O_2$  moves into the plane (positive y value)*

Before rotating  $O_1$  about  $C\alpha-C_1$ , rotate the frame  
by  $19.5^\circ$  about y:

$$\begin{aligned}
 Ry(19.5^\circ) \bullet \begin{bmatrix} 2.385 \\ 0 \\ 0.295 \end{bmatrix} &= \begin{bmatrix} \cos 19.5^\circ & 0 & -\sin 19.5^\circ \\ 0 & 1 & 0 \\ \sin 19.5^\circ & 0 & \cos 19.5^\circ \end{bmatrix} \begin{bmatrix} 2.385 \\ 0 \\ 0.295 \end{bmatrix} \\
 &= \begin{bmatrix} 2.248 + 0 - 0.098 \\ 0 + 0 + 0 \\ 0.796 + 0 + 0.278 \end{bmatrix} \\
 &= \begin{bmatrix} 2.15 \\ 0 \\ 1.074 \end{bmatrix}
 \end{aligned}$$

*Double-check:*

*In the new coordinate system, the  
z component must be much  
larger whereas x remains similar*

Now rotate about x' (C $\alpha$ -C<sub>1</sub>) by  $\psi$ :

$$R_x(\psi) \bullet \begin{bmatrix} 2.15 \\ 0 \\ 1.074 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & -\sin \psi \\ 0 & \sin \psi & \cos \psi \end{bmatrix} \begin{bmatrix} 2.15 \\ 0 \\ 1.074 \end{bmatrix}$$

$$= \begin{bmatrix} 2.15 + 0 + 0 \\ 0 + 0 - 1.074 \sin \psi \\ 0 + 0 + 1.074 \cos \psi \end{bmatrix}$$

$$= \begin{bmatrix} 2.15 \\ -1.074 \sin \psi \\ 1.074 \cos \psi \end{bmatrix}$$

*Double-check:*

*Based on dihedral angle definition, rotation by small positive  $\Psi$  means O<sub>1</sub> moves out of the plane (negative y value)*

Rotate the reference frame back by  $-19.5^\circ$   
about y to match the original coordinate system:

$$\begin{aligned}
 O_1' &= Ry(-19.5^\circ) \bullet \begin{bmatrix} 2.15 \\ -1.074 \sin \psi \\ 1.074 \cos \psi \end{bmatrix} = \begin{bmatrix} \cos(-19.5^\circ) & 0 & -\sin(-19.5^\circ) \\ 0 & 1 & 0 \\ \sin(-19.5^\circ) & 0 & \cos(-19.5^\circ) \end{bmatrix} \begin{bmatrix} 2.15 \\ -1.074 \sin \psi \\ 1.074 \cos \psi \end{bmatrix} \\
 &= \begin{bmatrix} 2.027 + 0 + 0.359 \cos \psi \\ 0 - 1.074 \sin \psi + 0 \\ -0.718 + 0 + 1.012 \cos \psi \end{bmatrix} \\
 &= \begin{bmatrix} 2.027 + 0.359 \cos \psi \\ -1.074 \sin \psi \\ -0.718 + 1.012 \cos \psi \end{bmatrix}
 \end{aligned}$$

To solve for unknown  $\phi$  and  $\psi$ , use the distance  
between  $O_1$  and  $O_2$

When  $\phi = \psi$ :

$$(2.217 \cos \phi - 2.027 - 0.359 \cos \phi)^2 + (2.217 \sin \phi + 1.074 \sin \phi)^2 + (1.51 + 0.718 - 1.012 \cos \phi)^2 = 2.7^2$$

$$(1.858 \cos \phi - 2.027)^2 + (3.291 \sin \phi)^2 + (2.228 - 1.012 \cos \phi)^2 = 7.29$$

$$3.452 \cos^2 \phi - 7.532 \cos \phi + 4.1087 + 10.831 \sin^2 \phi + 4.964 - 4.509 \cos \phi + 1.024 \cos^2 \phi - 7.29 = 0$$

$$\sin^2 + \cos^2 = 1 \Rightarrow$$

$$3.452 \cos^2 \phi - 7.532 \cos \phi + 4.1087 + 10.831 - 10.831 \cos^2 \phi + 4.964 - 4.509 \cos \phi + 1.024 \cos^2 \phi - 7.29 = 0$$

$$-6.355 \cos^2 \phi - 12.041 \cos \phi + 12.6137 = 0$$

$$6.355 \cos^2 \phi + 12.041 \cos \phi - 12.6137 = 0$$

$$\cos \phi = \frac{-12.041 \pm \sqrt{12.041^2 - 4(6.355)(-12.6137)}}{2(6.355)}$$

$$= \frac{-12.041 \pm \sqrt{144.986 - 320.64}}{12.71} = \frac{-12.041 \pm 21.578}{12.71}$$

$$= 0.750 \text{ or } -2.645 \text{ (ignored)}$$

invert cosine:

$$\phi = 41.4^\circ \quad \phi = -41.4^\circ$$

$$\psi = 41.4^\circ \quad \psi = -41.4^\circ$$

Likewise, when  $\phi = -\psi$

$$\phi = -73.975^\circ \quad \phi = 73.975^\circ$$

$$\psi = 73.975^\circ \quad \psi = -73.975^\circ$$



```

proc torus {} {
    set step 0.03
    set a 3
    set c 8

    for {set u 0} { $u < 6.2831853 } { set u [expr $u + $step]} {
        for {set v 0} { $v < 6.2831853 } { set v [expr $v + $step]} {

            set u1 [expr $step + $u]
            set v1 [expr $step + $v]

            set x0 [expr ($c+$a*cos($v))*cos($u)]
            set y0 [expr ($c+$a*cos($v))*sin($u)]
            set z0 [expr $a*sin($v)]

            set x1 [expr ($c+$a*cos($v))*cos($u1)]
            set y1 [expr ($c+$a*cos($v))*sin($u1)]
            set z1 [expr $a*sin($v)]

            set x2 [expr ($c+$a*cos($v1))*cos($u)]
            set y2 [expr ($c+$a*cos($v1))*sin($u)]
            set z2 [expr $a*sin($v1)]

            set x3 [expr ($c+$a*cos($v1))*cos($u1)]
            set y3 [expr ($c+$a*cos($v1))*sin($u1)]
            set z3 [expr $a*sin($v1)]

            draw triangle "$x0 $y0 $z0" "$x1 $y1 $z1" "$x2 $y2 $z2"
            draw triangle "$x2 $y2 $z2" "$x1 $y1 $z1" "$x3 $y3 $z3"
        }
    }
}

```

# Problem 3